Schwartz 8.5(a)

Pick frame $p = (E, 0, 0, p_z)$, then

$$\varepsilon_1^\mu = (0,1,0,0), \quad \varepsilon_2^\mu = (0,0,1,0), \quad \varepsilon_3^\mu = \left(\frac{p_z}{m}, 0, 0, \frac{E}{m}\right)$$

$$\sum_j \varepsilon_j^\mu \varepsilon_j^\nu = \begin{pmatrix} 0 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} + \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} + \frac{1}{m^2}\begin{pmatrix} p_z^2 & & & p_z E \\ & 0 & & \\ & & 0 & \\ p_z E & & & E^2 \end{pmatrix}$$

$$= \begin{pmatrix} p_z^2/m^2 & & & p_z E/m^2 \\ & 1 & & \\ & & 1 & \\ p_z E/m^2 & & & E^2/m^2 \end{pmatrix}$$

$$g_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad k_\mu k_\nu = \begin{pmatrix} E^2 & & & E p_z \\ & 0 & & \\ & & 0 & \\ E p_z & & & p_z^2 \end{pmatrix}$$

$$p_z^2/m^2 + 1 = \frac{1}{m^2}(p_z^2 + m^2) = \frac{1}{m^2} E^2 = E^2/m^2$$

$$E^2/m^2 - 1 = \frac{1}{m^2}(E^2 - m^2) = \frac{1}{m^2} p_z^2 = p_z^2/m^2.$$

This suggests
$$\begin{bmatrix} p_z^2/m^2 & & & p_z E/m^2 \\ & 1 & & \\ & & 1 & \\ p_z E/m^2 & & & E^2/m^2 \end{bmatrix} = \begin{bmatrix} \frac{E^2}{m^2} - 1 & & & \frac{E p_z}{m^2} \\ & +1 & & \\ & & +1 & \\ \frac{E p_z}{m^2} & & & \frac{p_z^2}{m^2} + 1 \end{bmatrix}$$

$$= \boxed{\frac{k_\mu k_\nu}{m^2} - g_{\mu\nu}}$$

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3.9.2024

(b) We have just found $\sum_j \varepsilon_j^\mu \varepsilon_j^\nu = \frac{k_\mu k_\nu}{m^2} - g_{\mu\nu}$

The previous solution I found for the massive spin-1 propagator was

$$\Pi_{\mu\nu} = \frac{k_\mu k_\nu}{k^2 m^2}$$

Putting it in the conventional form of $\Pi = \frac{(\cdots)}{k^2 - m^2 + i\varepsilon}$,

we have $\Pi_{\mu\nu} = \frac{k_\mu k_\nu}{k^2 m^2} \frac{k^2 - m^2}{k^2 - m^2}$

$$= \frac{\frac{k_\mu k_\nu}{k^2 m^2}(k^2 - m^2)}{k^2 - m^2 + i\varepsilon}$$

← numerator that we are interested in

$$\frac{k_\mu k_\nu}{k^2 m^2}(k^2 - m^2) = \frac{k_\mu k_\nu}{\cancel{k^2} m^2}\cancel{k^2} - \frac{k_\mu k_\nu}{k^2 \cancel{m^2}}\cancel{m^2}$$

$$= \frac{k_\mu k_\nu}{m^2} - \frac{k_\mu k_\nu}{k^2} = \boxed{\frac{k_\mu k_\nu}{m^2} - g_{\mu\nu}}$$

⇑

~~This matches.~~

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